1 Limits of Polynomial Functions

Theorem. If p is a polynomial function and c is a real number, then

$$\lim_{x \to c} p(x) = p(c)$$

Let's explore this theorem by examining the limits of several polynomial functions in order of increasing complexity.

Example 1. Find the limit and support your result by graphing the function and constructing a table of data.

$$\lim_{x \to 1} 3$$

Solution Since $p(x) = 3$ is a polynomial function and more specifically a constant function, by the theorem

$$\lim_{x \to 1} 3 = p(1) = 3.$$

Evaluate the function at values very close to 1 that are less than and greater than 1. By constructing a table similar to the one below, the limit is estimated to be 3.

$\boldsymbol{x}$	0.900	0.990	0.999	1.000	1.001	1.010	1.100
$\boldsymbol{p(x)}$	3	3	3	?	3	3	3

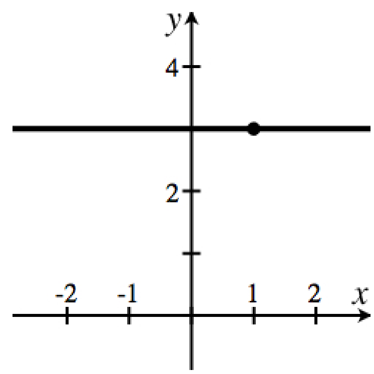

Figure 1: The limit as x approaches 1 is 3.

Example 2. Find the limit and support your result by graphing the function and constructing a table of data.

$$\lim_{z \to 1} (2z - 4)$$

Solution Since $p(z) = 2z - 4$ is a polynomial function and more specifically a linear function, by the theorem

$$\lim_{z \to 1} (2z - 4) = p(1) = 2(1) - 4 = -2.$$

Evaluate the function at values very close to 1 that are less than and greater than 1. By constructing a table similar to the one below, the limit is estimated to be -2.

$\boldsymbol{z}$	0.900	0.990	0.999	1.000	1.001	1.010	1.100
$\boldsymbol{p(z)}$	-2.2	-2.02	-2.002	?	-1.998	-1.98	-1.8

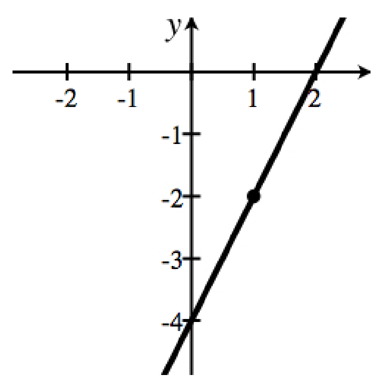

Figure 2: The limit as z approaches 1 is -2.

Example 3. Find the limit and support your result by graphing the function and constructing a table of data.

$$\lim_{y \to -1} (-y^2 + 3y - 2)$$

Solution Since $p(y) = -y^2 + 3y - 2$ is a polynomial function and more specifically a quadratic function, by the theorem

$$\lim_{y \to -1} (-y^2 + 3y - 2) = p(-1) = -(-1)^2 + 3(-1) - 2 = -6.$$

Evaluate the function at values very close to -1 that are less than and greater than -1. By constructing a table similar to the one below, the limit is estimated to be -6.

y	-1.100	-1.010	-1.001	-1.000	-0.999	-0.990	-0.900
$p(y)$	-6.51	-6.050	-6.005	?	-5.995	-5.950	–5.51

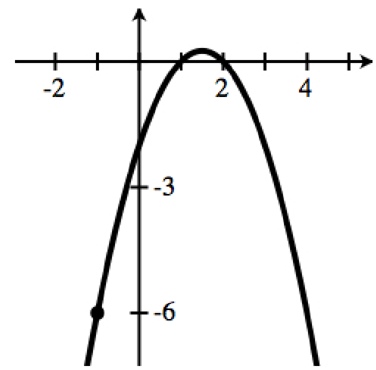

Figure 3: The limit as y approaches -1 is -6.

Example 4. Find the limit and support your result by graphing the function and constructing a table of data.

$$\lim_{z \to -2} (z^3 + 3z^2 + z - 2)$$

Solution Since $p(z) = z^3 + 3z^2 + z - 2$ is a polynomial function and more specifically a cubic function, by the theorem

$$\lim_{z \to -2} (z^3 + 3z^2 + z - 2) = p(-2) = (-2)^3 + 3(-2)^2 + (-2) - 2 = 0.$$

Evaluate the function at values very close to -2 that are less than and greater than -2. By constructing a table similar to the one below, the limit is estimated to be 0.

z	-2.100	-2.010	-2.001	-2.000	-1.999	-1.990	-1.900
$p(z)$	-0.131	-0.0103	-0.0010	?	0.0010	0.0097	0.071

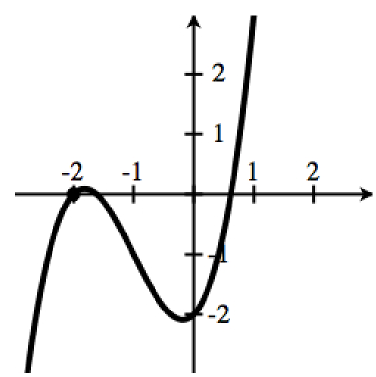

Figure 4: The limit as z approaches -2 is 0.

Example 5. Find the limit and support your result by graphing the function and constructing a table of data.

$$\lim_{y \to 0}(y^{99} - 60y^{73} - 12y^{17} + 7y^5 + 3y + 1)$$

Solution Since $p(y) = y^{99} - 60y^{73} - 12y^{17} + 7y^5 + 3y + 1$ is a polynomial function, by the theorem

$$\lim_{y \to 0}(y^{99} - 60y^{73} - 12y^{17} + 7y^5 + 3y + 1) = p(0)$$
$$= (0)^{99} - 60(0)^{73} - 12(0)^{17} + 7(0)^5 + 3(0) + 1 = 1.$$

Evaluate the function at values very close to 0 that are less than and greater than 0. By constructing a table similar to the one below, the limit is estimated to be 1.

$\boldsymbol{y}$	-0.100	-0.010	-0.001	0.000	0.001	0.010	0.100
$\boldsymbol{p(y)}$	0.700	0.97	0.997	?	1.003	1.03	1.300

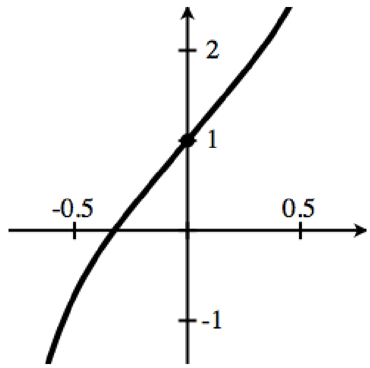

Figure 5: The limit as y approaches 0 is 1.